

Measurement of the Orbital Angular Momentum Spectrum of Partially Coherent Fields using Double Angular Slit Interference

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We implement an interferometric method using two angular slits to measure the orbital angular momentum (OAM) mode spectrum of a partially coherent field. As the angular separation of the slits changes, an interference pattern for a particular OAM mode is obtained. The visibility of this interference pattern as a function of angular separation is equivalent to the angular correlation function of the field. By Fourier transforming the angular correlation function obtained from the double angular slit interference, we are able to calculate the OAM spectrum of the partially coherent field. This method has potential application for characterizing the OAM spectrum in high-dimensional quantum information protocols.

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I. INTRODUCTION

Historically, the problem of determining the correlations between two space-time points in a wave field has been addressed by the analysis of a two-beam interference experiment [1, 2]. The degree of spatial coherence of a light source can be determined from the visibility of fringes in the far-field, arising from the interference of light fields in a Young's double slit experiment. The necessary condition for interference is that the slit separation should be smaller than the transverse coherence length of the field.

It is well known that angular position and its conjugate variable, orbital angular momentum (OAM), form Fourier transform pairs [3]. This is convincingly demonstrated by the experimental observation of a discrete diffraction pattern in the OAM spectrum when light is transmitted through angular amplitude masks [4, 5]. Extending this idea, one can construct an angular analog of Young's double slit experiment with two angular slits [6]. By measuring the visibility of the interference pattern obtained from double angular slit interference, one can calculate the degree of angular coherence of the source. For certain types of partially coherent sources, the OAM mode spectrum is simply a Fourier transform of the degree of angular coherence [6]. This provides us with a new way of measuring the OAM mode spectrum of partially coherent fields.

Interest in the measurement and characterization of the OAM mode spectrum has grown rapidly over the last decade. This is because OAM modes, by virtue of living in a discrete, infinite-dimensional Hilbert space, are a very useful tool for quantum information science. Entanglement in up to 12 dimensions has been shown using OAM modes [7]. Efforts are under way to realize a quantum key distribution system using the OAM basis and the conjugate basis of angular position [8, 9]. The simplest way of measuring the OAM mode spectrum is by

projection measurements of the constituent modes using a forked hologram [10]. More novel methods have been developed recently that involve interfering the field with a rotated copy of itself in a Mach-Zehnder interferometer [11, 12].

In this paper, we experimentally demonstrate a new way of measuring the OAM mode spectrum of partially coherent fields. Based on the theoretical work in Ref. [6], we first review the concepts of the degree of angular coherence and double angular slit interference. Then, we describe our experimental implementation of double angular slit interference using a spatial light modulator. Finally, we calculate the OAM mode spectrum from the visibility of the measured angular interference pattern.

II. THEORY

The cross-spectral-density function is used to quantify field correlations between two points in the space-frequency domain. The non-negative definiteness of this function, in conjunction with the Mercer theorem, imply that it is Hilbert-Schmidt kernel and that it has a

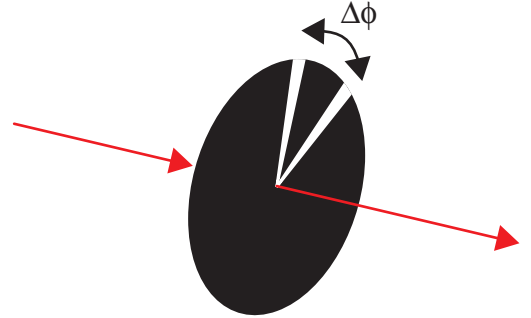


FIG. 1: Partially coherent field propagating through double angular slits with an angular separation of $\Delta\phi$.

coherent-mode representation of the form [13]

$$W(\mathbf{r}_1, \mathbf{r}_2) = \sum_n \alpha_n \psi_n^*(\mathbf{r}_1) \psi_n(\mathbf{r}_2) \quad (1)$$

The properties of $W(\mathbf{r}_1, \mathbf{r}_2)$ ensure that all the coefficients α_n are real and non-negative, i.e. $\alpha_n \geq 0$, and that at least one α_n is non-zero. This further implies that for any partially coherent field, there exists at least one basis in which the cross-spectral-density function can be represented as the superposition of modes that are completely coherent in the space-frequency domain.

For fields in the Laguerre-Gauss (LG) basis, one can define a cross-spectral-density function in the angle-frequency domain by integrating over the radial dimension [6]. The cross-spectral-density function can then be expressed as a superposition of OAM modes

$$W(\phi_1, \phi_2) = \sum_{\ell=-\infty}^{\infty} C_{\ell} e^{-i\ell\Delta\phi} \quad (2)$$

If there is only one term in the above expansion, the field can be considered to be completely coherent in the angle-frequency domain. If there is more than one term in the expansion, the field can be characterized as being partially coherent. For such fields, one can introduce the angular analog to coherence length, the coherence angle, ϕ_{coh} . The coherence angle quantifies the angular separation over which two field points are mutually coherent.

These concepts are best illustrated by the case of angular interference from a double angular slit (Fig. 1). When a partially coherent field is transmitted through a mask containing two angular slits at angular positions ϕ_1 and ϕ_2 with unit transmission, an interference pattern is obtained with intensity in an OAM mode ℓ given by [6]

$$I_{\ell}(\Delta\phi) = \frac{1}{\pi} \sum_{\ell'=-\infty}^{\infty} C_{\ell'} [1 + \lambda(\Delta\phi) \cos(\ell\Delta\phi)]. \quad (3)$$

Here, $\lambda(\Delta\phi)$ is the degree of angular coherence and is equal to

$$\lambda(\Delta\phi) = \frac{W(\phi_1, \phi_2)}{\sum_{\ell'=-\infty}^{\infty} C_{\ell'}} \quad (4)$$

where $\Delta\phi = \phi_1 - \phi_2$. Eq. 3 is the angular interference law, since it quantifies the interference between the fields coming from two separate angular positions. This is analogous to the general interference law for stationary optical fields [1]. Also, one can see that $\lambda(\Delta\phi)$ is equal to the visibility of the interference pattern, which is a function of the angular slit separation. The width of $\lambda(\Delta\phi)$

is a measure of the coherence angle ϕ_{coh} over which the field remains coherent. For a completely coherent field, the degree of angular coherence $\lambda(\Delta\phi)$ is equal to unity for all values of $\Delta\phi$.

It has been shown in Ref. [6] that for partially coherent fields with a broad Gaussian distribution in ℓ , the angular correlation function is simply the Fourier transform of the OAM mode distribution. More specifically, for a Gaussian OAM mode distribution with a width σ ,

$$C(\ell) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(\frac{-\ell^2}{2\sigma^2}\right). \quad (5)$$

For such a distribution of OAM, the degree of angular coherence is given by

$$\lambda(\Delta\phi) = \exp\left(\frac{-\sigma^2\Delta\phi^2}{2}\right). \quad (6)$$

By measuring the visibility of the interference pattern as a function of slit separation $\Delta\phi$, we can construct the degree of coherence $\lambda(\Delta\phi)$ of the field. The width of the visibility function, $1/\sigma$, gives us the coherence angle of the field, ϕ_{coh} . From this, we can calculate σ , which is the width of the OAM mode spectrum of the partially coherent field, and reconstruct the OAM mode distribution from Eq. (5).

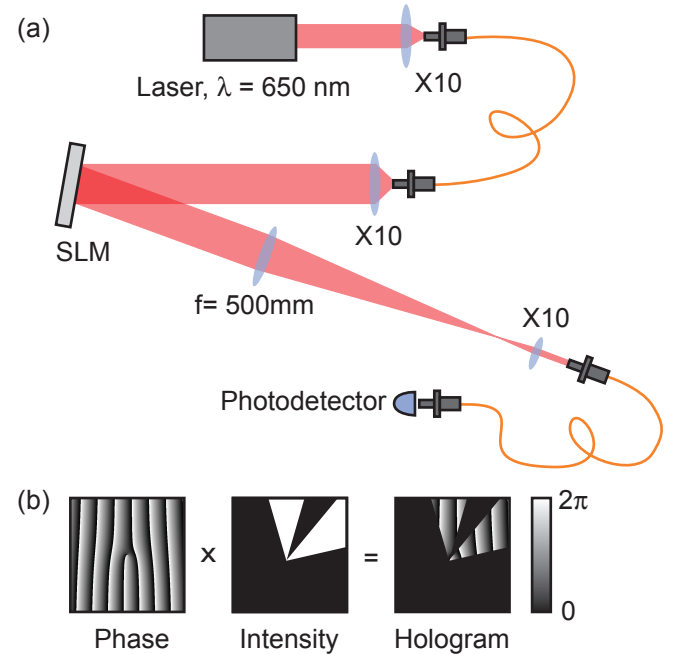


FIG. 2: (a) Schematic of the experimental setup (b) Example of a phase hologram impressed on the SLM

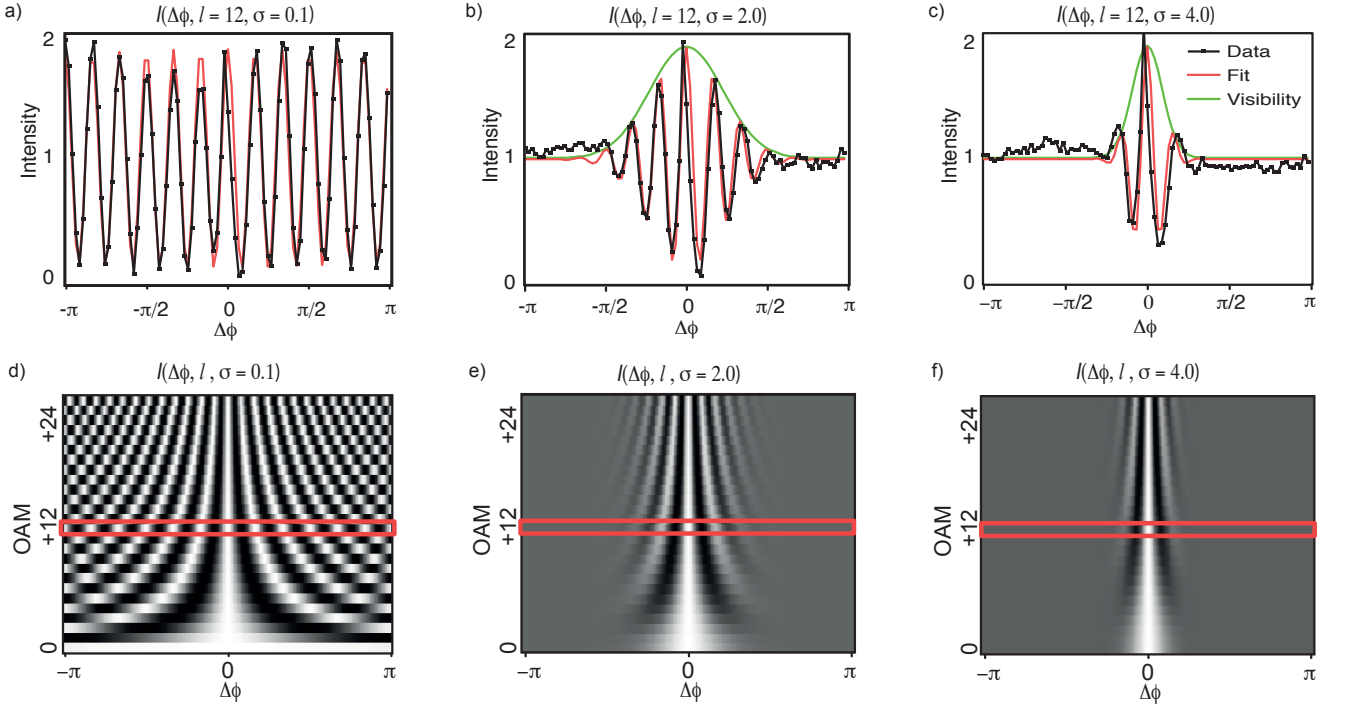


FIG. 3: Intensity I_ℓ , for OAM mode projection $\ell=12$, of partially coherent light transmitted through identical angular apertures, as a function of the angular separation $\Delta\phi$ for different OAM mode distribution widths (a) $\sigma = 0.1$ (b) $\sigma = 2$ and (c) $\sigma = 4$. Experimental results (black line) are compared with simulation results (red line) obtained from theory. A fit to the visibility envelope as a function of angular slit separation $\Delta\phi$ is plotted (green line). (d)-(f) Simulation results for angular interference patterns for OAM mode projections from $\ell = 0$ to $\ell = 24$ for the three values of σ . The red box indicates the value of ℓ used in our experiment.

III. EXPERIMENT AND RESULTS

We implemented a double angular slit experiment using a setup shown in Fig. 2(a). Multimodal light from a laser diode at 650nm is coupled into a single mode fiber (SMF) with a 10x microscope objective. The Gaussian single mode output from the SMF is collimated and is incident on the front surface of a spatial light modulator (SLM). It is a simple task to use an SLM to generate a coherent superposition of OAM modes; however, for this experiment, we require an incoherent Gaussian distribution of OAM modes with a width given by σ . To generate the incoherent superposition, we generate a coherent superposition of OAM modes with a single hologram. The range of OAM modes that we generate at this stage has the required width σ , but each of the modes in the superposition is given a random phase. We then generate a different superposition of OAM modes using a new hologram in which each of the modes is given a new random phase. Changing the holograms in this way ensures that there is no phase relationship between the modes generated by subsequent holograms. This procedure is repeated over a thousand times such that the resulting time-averaged beam that reflects off the SLM is a partially coherent superposition of OAM modes with a width

given by σ .

For every incoherent Gaussian distribution of OAM modes with a width given by σ , the SLM is then used to (i) generate a forked phase hologram to project the field into an OAM mode with angular momentum ℓ , and (ii) generate an amplitude mask of two angular slits with 15° widths, with one slit fixed at 0° and the other rotating from $-\pi$ to π radians. An example of the phase and amplitude pattern impressed on the SLM is given in Fig. 2(b).

The light from the SLM is imaged onto the aperture of a SMF connected to a photodetector. In our experiment, we chose to measure the interference pattern given by Eq. (3) by projecting the field into a mode with $\ell = 12$. The intensity at the photodetector as a function of the angular separation $\Delta\phi$ between the angular slits is shown in Fig. 3 (a)-(c). For a completely coherent field ($\sigma = 0.1$), there is only a single OAM mode $\ell = 0$ in the generated field and the degree of coherence $\lambda(\Delta\phi) \approx 1$. The forked hologram with $\ell = 12$ projects the field into an OAM mode with $\ell = -12$. Eq. (3) then simplifies to the form

$$I(\Delta\phi) = \frac{1}{\pi} [1 + \cos(12\Delta\phi)]. \quad (7)$$

From this, we can see that the interference pattern thus obtained will have 12 fringes. This can be interpreted as there being 12 angular positions of the second angular slit as it rotates from $-\pi$ to π radians, for which the transmitted fields through the two angular slits are exactly in phase. This gives rise to the 12 fringes seen in the intensity plot in Fig. 3(a) (plotted in black). Since the field has only one OAM mode and is completely coherent in angular position, the visibility envelope of the fringe pattern is close to one for all values of angular separation. When $\sigma = 2$, there are additional OAM modes in the input Gaussian distribution. It is evident from Fig. 3(b) that the width of the visibility envelope is significantly reduced and only 5 fringes are detected. This is due to the fact that the angular width over which the field remains mutually coherent, or the coherence angle ϕ_{coh} , is reduced. It can be seen from Fig. 3 (a)-(c) that as the width σ of the OAM-mode distribution increases, the coherence angle further decreases and fewer interference fringes are visible as $\Delta\phi$ goes from $-\pi$ to π radians.

Simulation results for angular interference patterns for OAM mode projections from $\ell = 0$ to $\ell = 24$ are plotted in Fig. 3 (d)-(f). As can be seen, the width of the visibility envelope ($\lambda(\Delta\phi)$) is independent of the OAM mode number used. The simulated interference pattern obtained by projecting in OAM mode $\ell = 12$ is outlined in red and is also plotted in red in Fig. 3 (a)-(c) as a fit to the experimental data.

When the input OAM mode distribution has a broad gaussian shape, we can plot the visibility envelope of the interference pattern using Eq. (6). This condition is held for the case of $\sigma = 2.0$ and $\sigma = 4.0$. In Fig. 3 (b) and (c), we plot a fit to the visibility envelope of the angular interference pattern for these two cases (plotted in green). The width of the visibility curve gives us the value of the coherence angle, ϕ_{coh} . The measured value of ϕ_{coh} is 0.71 and 0.29 radians for an input OAM mode distribution width of $\sigma = 2.0$ and 4.0 respectively. From Eqs. (5) and (6), we see that the width of the OAM mode distribution is equal to the inverse of the coherence angle. Using this, we calculate the measured OAM mode distribution width $\sigma_m = 1.4$ and 3.4 for input widths of $\sigma = 2.0$ and 4.0 respectively. The measured OAM mode distributions for these two cases are plotted in Fig. 4. The discrepancy between input mode distribution width and measured mode distribution width is most likely due to lack of precise control on the width of the partially coherent input OAM mode distribution.

IV. CONCLUSION

We have demonstrated experimental results to support the theoretical analysis of angular coherence proposed in Ref. [6] for a partially coherent light source. We employed an SLM and a classical light source to cre-

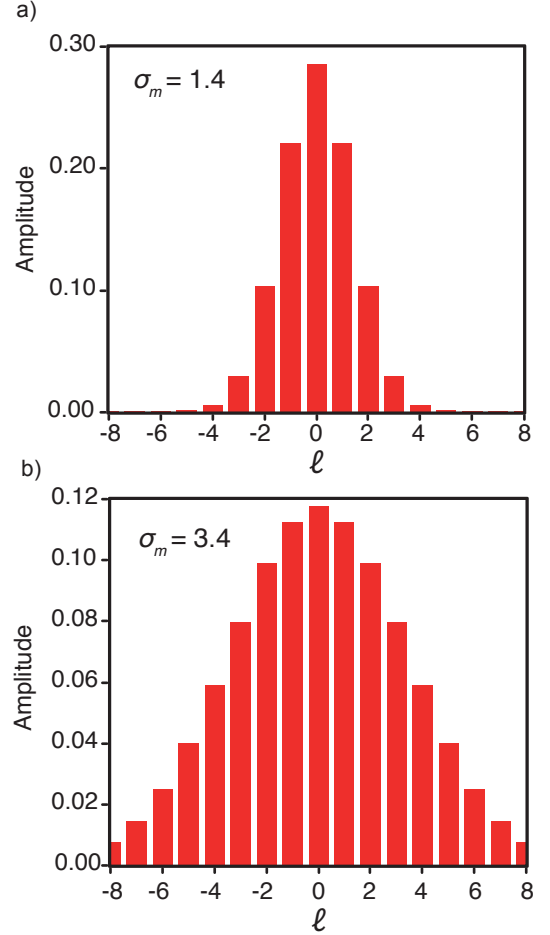


FIG. 4: Calculated OAM mode spectrum for the two cases of measured OAM distribution widths of (a) $\sigma_m = 1.4$ and (b) $\sigma_m = 3.4$. The corresponding input OAM distribution widths are $\sigma = 2.0$ and $\sigma = 4.0$ respectively.

ate a partially coherent Gaussian distribution of OAM modes. When this beam was passed through a double angular-slit, a characteristic interference pattern consisting of fringes inside a visibility envelope was obtained for a specific OAM mode projection. The width of this visibility envelope served as a direct measure of the angular coherence of the source, and allowed us to reconstruct the input OAM mode distribution to a good approximation. This was consistent with our simulation results which were modeled on the theory of partial angular coherence put forth in Ref. [6]. We envision that such measurements will be useful for quantum information protocols that rely on the high-dimensionality of the OAM state space [8, 14]. As these protocols are usually implemented in free-space communication systems [9], it is important to characterize the effects of turbulence on the angular coherence of such a system. Additionally, it has been shown in Ref. [6] that such a measurement could be carried out on either the signal or idler arm in order to obtain the angular coherence properties of

the entangled two-photon field produced by spontaneous parametric down conversion [10]. We expect that angular interference from a double angular-slit holds promise for exploring many interesting quantum phenomena in the high-dimensional space of orbital angular momentum.

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